Chapter IV

Applications of Gravity Thrust Trajectories in Strategic Interplanetary Mission Design

4.1 The Initial Mass Problem of Rocket Engines

It is convenient to describe energy requirements for various interplanetary missions by a parameter called "Characteristic Velocity." We shall denote this parameter by V_c . It is the sum total of all changes in velocity magnitudes which is required by the vehicle for a particular mission. Thus, $V_c = \sum_{i=1}^{n} \Delta V_i$, where n is the number of times rocket thrust is applied. Interplanetary missions are usually viewed as originating from an earth "parking orbit." A mission begins by injecting the vehicle out of its parking orbit and onto its initial escape trajectory. To achieve this injection, the vehicle increases its orbital velocity by an amount ΔV_1 .

One of the most important equations in the field of space flight is the so called Mass-Ratio Equation (also known as the Rocket Equation). Let c denote the exhaust velocity of a rocket engine on board a space vehicle moving in a free-fall state. If we require the vehicle to change its velocity by a magnitude ΔV , then a certain amount of its mass in the form of rocket fuel will have to be expelled by the engine. If m_1 and m_2 denote the total vehicle mass before and after achieving the velocity change respectively, the Mass-Ratio equation states that

$$\frac{m_2}{m_1} = \exp\left(- \frac{\Delta V}{c} \right) . \tag{4.1.1}$$

It is thermodynamically impossible to construct an ordinary chemical rocket engine which will generate exhaust velocities greater than approximately 4.4 km/sec. Nuclear propulsion engines are being developed which may yield exhaust velocities between 8 and 9 km/sec. Exhaust velocities in excess of 10 km/sec will require operating temperatures so high that basic problems in fields such as metallurgy will be extremely difficult to solve and will require major scientific breakthroughs. With definite upper bounds on c, it is clear from the Mass-Ratio equation that any ambitious interplanetary mission requiring V_c's of more than 15 km/sec will be extremely difficult to achieve with current rocket technology. For example, omitting the calculations, it is easy to show that to soft land a simple 450 kgm. instrument package on the surface of Mercury via a direct Earth-Mercury transfer will require an initial orbital mass of about 62,000 kgm (using the most powerful chemical rocket engines with $c \approx 4.2$ km/sec). To send a 450 kgm instrumented probe to within one solar radius of the Sun would require a fantastic initial orbital mass of about 140,000 kgm. Studies (10) have shown that to carry out a modest, one year, round trip, five-man landing expedition to Mars using presently envisioned nuclear rocket propulsion and conventional Earth-Mars, Mars-Earth transfers would require two 450,000 kgm vehicles already in orbit.

Most of the initial mass of these vehicles is fuel. This of course is particularly true of vehicles using chemical rocket propulsion. Hence, the major problem involved in extended interplanetary space travel lies with the rocket engine. Since all rocket engines derive their thrust by expelling mass, they are limited as to how much total thrust they can deliver and how long they can operate. I call this characteristic of rocket engines the initial mass problem. Vehicle staging does help but if a mission's characteristic velocity is high,staging can not significantly reduce the required initial mass.

4.2 Unmanned Exploration of the Solar System

My first preliminary numerical calculations of Gravity Thrust trajectories were carried out in December 1961, on a small IBM 1620 digital computer at the California Institute of Technology's Jet Propulsion Laboratory. Two rather significant results were obtained: First, I discovered that even relatively low mass planets like Venus could be used to radically change a free-fall space vehicle's orbit about the Sun. (This fact is important because Venus can be reached by rather low energy initial transfer trajectories.) Secondly, I discovered that an IBM 1620 digital computer is several orders of magnitude too small for any extensive numerical study of Gravity Thrust trajectories. Consequently, in February 1962, I began an extensive study at UCLA on their recently acquired IBM 7090 computer (11). The study was expanded June 1962 by also using the large computing complex at J.P.L. What seemed to be an interesting theoretical game of interplanetary billiards in 1961 suddenly began to appear as the means for achieving significant energy reductions for almost all types of interplanetary missions. Much of this early numerical research was focused on the inner planets Mercury, Venus, Earth, and Mars. I discovered that many new interplanetary missions were within reach of launch vehicles in use or being developed at that time. Even manned missions to Mars appeared possible using only Saturn V type booster vehicles (9). My numerical calculations involving the outer planets was differed until an adequate planetary ephemeris extending to the year 2000 was obtained. However, by that time the study essentially took on the form of data processing. The theoretical questions were settled August 1961. The data processing of trajectories discussed in Chapter 3 was completed August 1964 (12).

In the remainder of this chapter I shall illustrate how Gravity Thrust space trajectories can be applied to specific missions. The greatest immediate use of Gravity Thrust trajectories lies in the unmanned exploration of the solar system. Direct transfer trajectories to Mercury of the form Earth-Mercury require (on the average) a minimum departing hyperbolic excess velocity (HEV) of approximately 7 km/sec (with flight times of 90 to 140 days). However, Venus can be reached with a minimum of only 3 km/sec. Then after reaching Venus, the vehicle could be redirected by gravity thrust onto another interplanetary trajectory which will enable it to rendezvous with Mercury. The implications of this possibility become strikingly clear when we substitute these numbers into the mass ratio equation. For example, if the vehicle's initial mass is 2,000 kgm, then the maximum allowable payload mass for a vehicle on a direct Earth-Mercury trajectory, using the most powerful chemical rocket engine ($c = 4.2$ km/sec), would be $2000 \times \exp(-7/4.2) = 378$ kgm. On the other hand, the vehicle's payload on an Earth-Venus-Mercury profile (for the same initial mass) would be $2000 \times \exp(-3/4.2) = 833$ kgm, an increase of 455 kgm or more than double the payload. The total flight times required

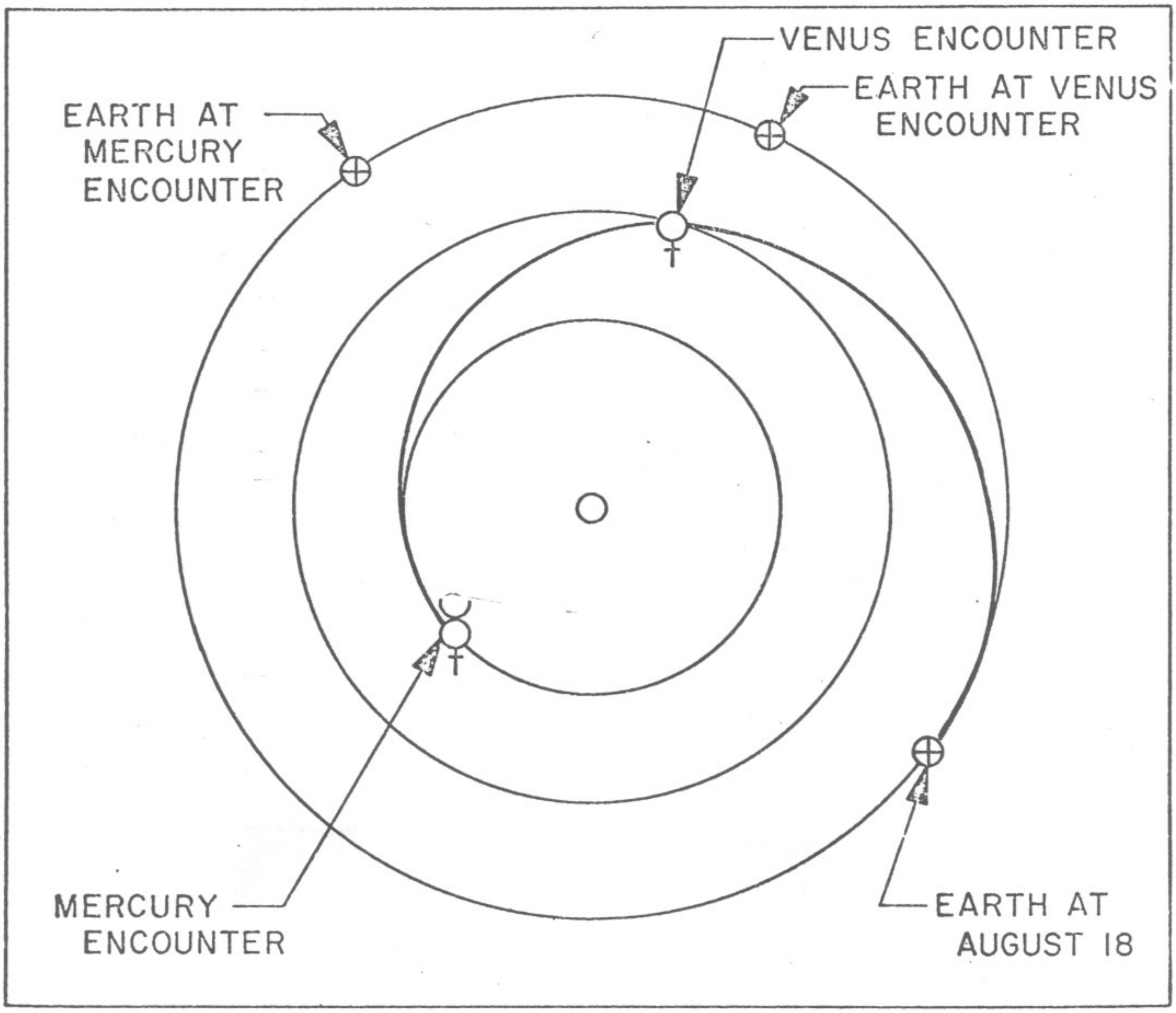

Figure 10. Trajectory profile for Earth-Venus-Mercury 1970. The distance of closest approach is approximately 4,000 Km. These profiles occur almost every time trips to Venus are possible (i.e., every 580 days).

to reach Mercury on an Earth-Venus-Mercury profile vary from 130 to 300 days. The extra benefit of passing Venus enroute to Mercury add to the attractiveness of these profiles. An example is shown in figure 10.

Since the orbits of most planets in our solar system are approximately circular and co-planar, the miminum energy, direct transfer trajectories P_0-P_1 , closely resembles Hohmann transfers. Hence, the length of time separating two successive minimum energy launch periods is approximately equal to the two planet's synodic period. Unfortunately, the synodic period for Earth and Mars is 780 days. Thus, although the required minimum energies for direct transfer trajectories Earth-Mars are approximately equal to those of Earth-Venus, these launch opportunities are separated by 780 days. However, it is also possible to reach Mars via Earth-Venus-Mars profiles. When these profiles are added to the conventional Earth-Mars transfers, the total number of launch opportunities for trips to Mars is almost doubled. The flight times for direct Earth-Mars transfers run from 200 to 350 days. The Earth-Venus-Mars profiles require from 250 to 500 days, using launch energies only slightly higher than the minimum energy Earth-Venus transfers. An example is shown in figure 11.

The Venus gravitational field could also be used to send unmanned instrumented probes to within .25 A.U. of the Sun. Such probes would be useful in measuring environmental conditions in regions relatively close to the Sun. The

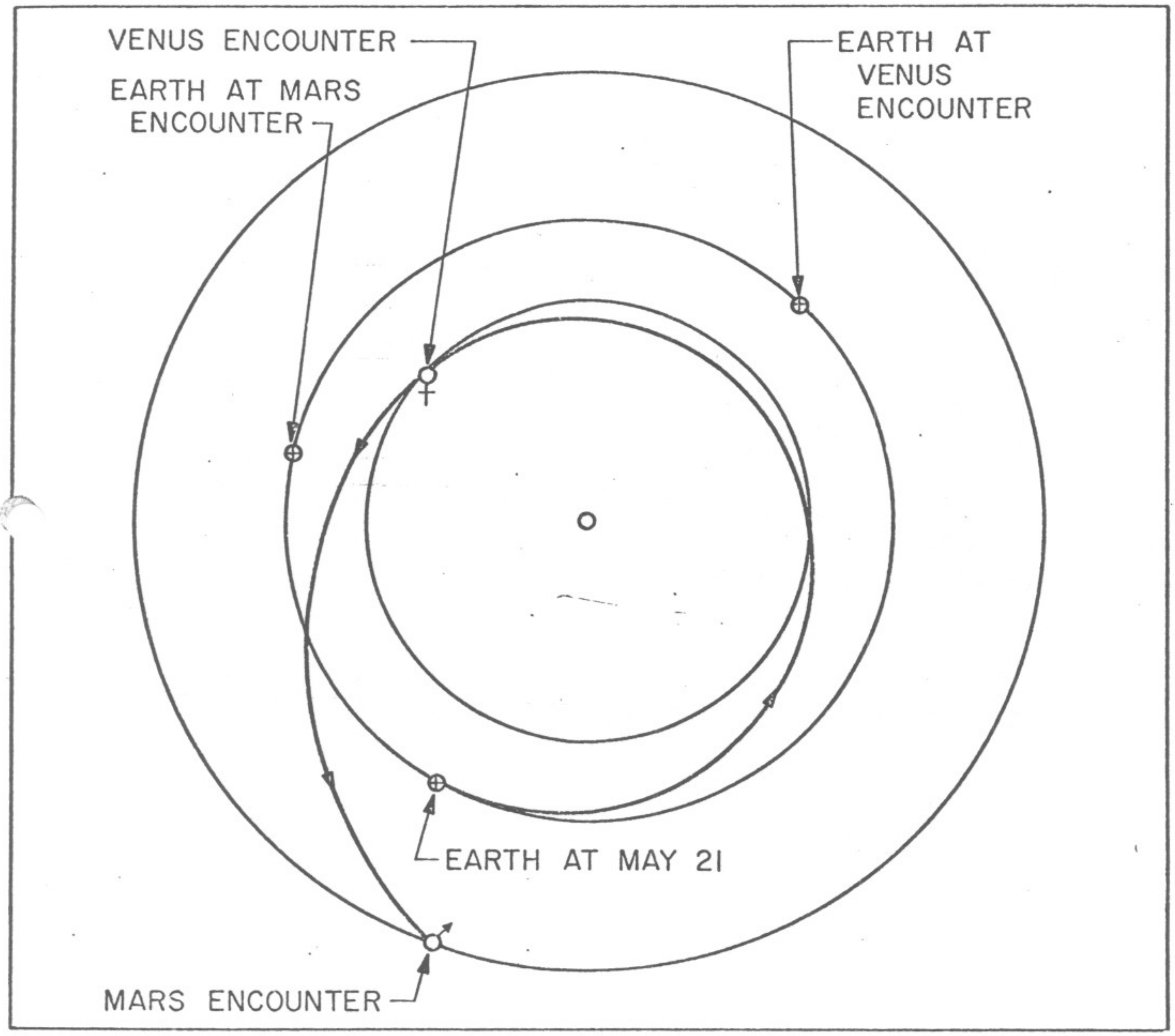

Figure 11. Trajectory profile for Earth-Venus-Mars 1972. The distance of closest approach is approximately 1,000 Km. Four of these opportunities exist on the average during each decade.

required minimum HEV's to reach these regions directly from Earth is approximately 11 km/sec. They could be reached with HEV's of only about 7 km/sec, using Earth-Venus profiles. This would double the possible payload.

Finally, the Venus field could also be used to achieve 12° out of ecliptic orbits. This would allow a vehicle's payload to be more than doubled. A near minimum energy transfer to Mars can be used to reach distances of from .7 to 2.0 A.U. from the Sun.

The above profiles show how it is possible to significantly increase the unmanned exploration of the inner solar system by using Gravity Thrust trajectories involving Venus and Mars. I shall now show how it is possible to carry out a complete unmanned exploration of the entire solar system using Gravity Thrust trajectories involving the outer planets.

The exploration of regions extremely close to the Sun's surface may be useful in obtaining a complete study of the interplanetary environment. A specially constructed, heat resistant probe with its own elaborate internal cooling system could obtain valuable information on Sun spots, solar flares, and solar magnetic fields. Unfortunately, to achieve a solar impact trajectory by a direct launch from Earth requires the complete cancellation of the Earth's orbital speed about the Sun. Hence, since the Earth's orbital speed is 29.8 km/sec, these direct transfer trajectories would require HEV's of 29.8 km/sec. Solar impact trajectories can also be obtained via Gravity Thrust from simple minimum energy

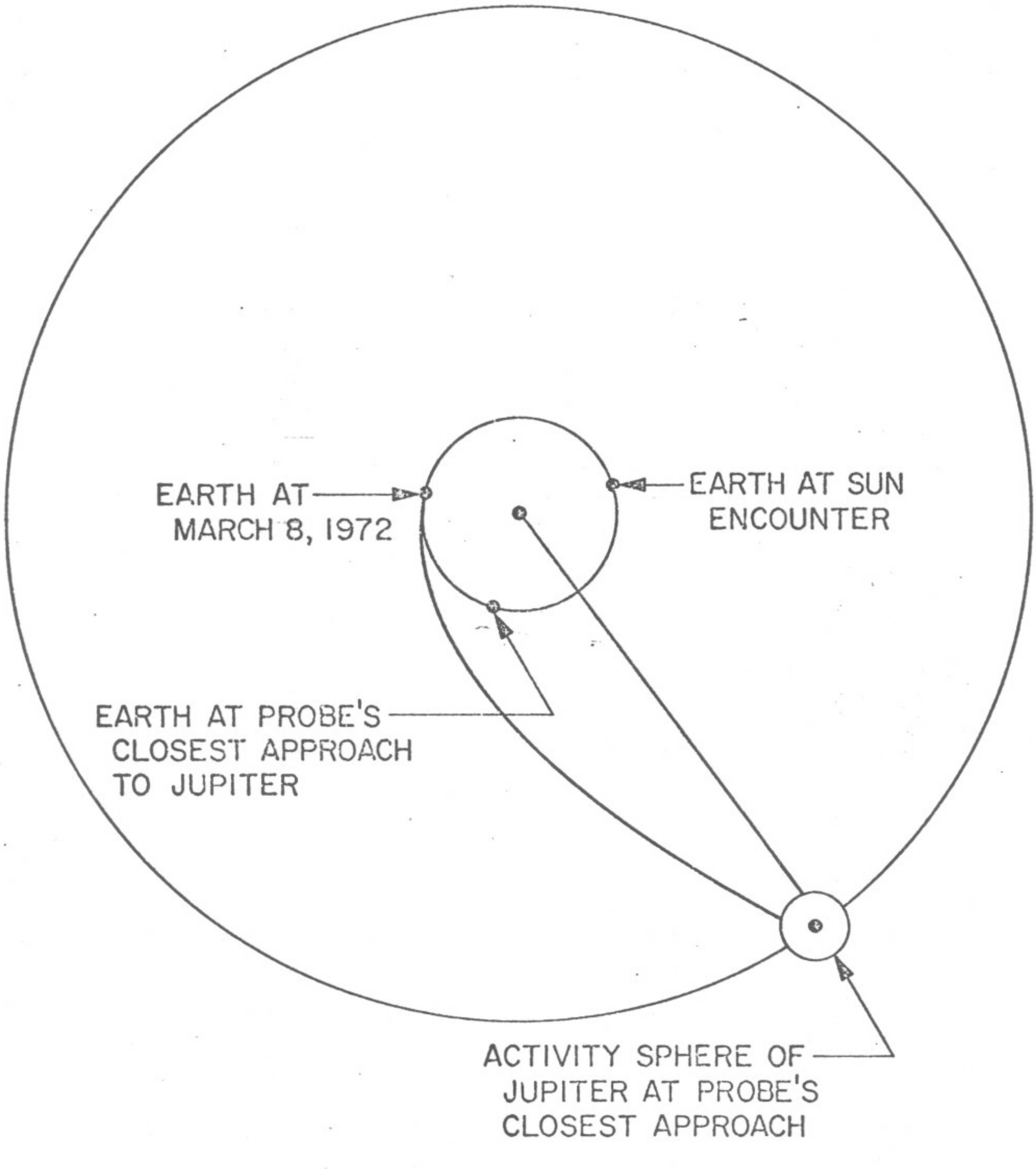

Figure 12. Trajectory profile for Earth-Jupiter-Sun 1971. The distance of closest approach is approximately 130,000 Km.

Earth-Jupiter transfers. Since these transfers require HEV's of only about 9.3 km/sec, the energy savings will be quite substantial. For example, if the vehicle's payload mass is fixed at 500 kgm., then the required initial mass for a classical direct transfer trajectory would come to 600,000 kgm. The same payload can also be delivered from an indirect Earth-Jupiter route with an initial mass of only 4,600 kgm. This represents a reduction by a factor of 130. On a cost basis of \$20/kgm, this translates into a savings of $\$1.2 \times 10^7$. The total flight time from launch to solar impact is about 3.5 years. These profiles would allow the probe to continuously monitor the interplanetary environment all the way from 5.0 A.U. right up to the Sun's surface. An example is given in figure 12. These opportunities occur during every minimum energy Earth-Jupiter launch period. Two successive launch periods are separated by approximately 400 days (the synodic period for Earth and Jupiter).

The exploration of regions far from the Sun can also be economically carried out by using Gravity Thrust. High energy direct transfer trajectories into deep space, escaping the entire solar system, with HEV's (Sun) of 20 kgm/sec will require launch HEV's (Earth) of about 17 km/sec. The same solar escape trajectories can be accomplished via Earth-Jupiter transfers with launch HEV's (Earth) of only about 11 km/sec. This corresponds to an initial mass reduction by a factor of 4.2. An example is given in figure 13. The

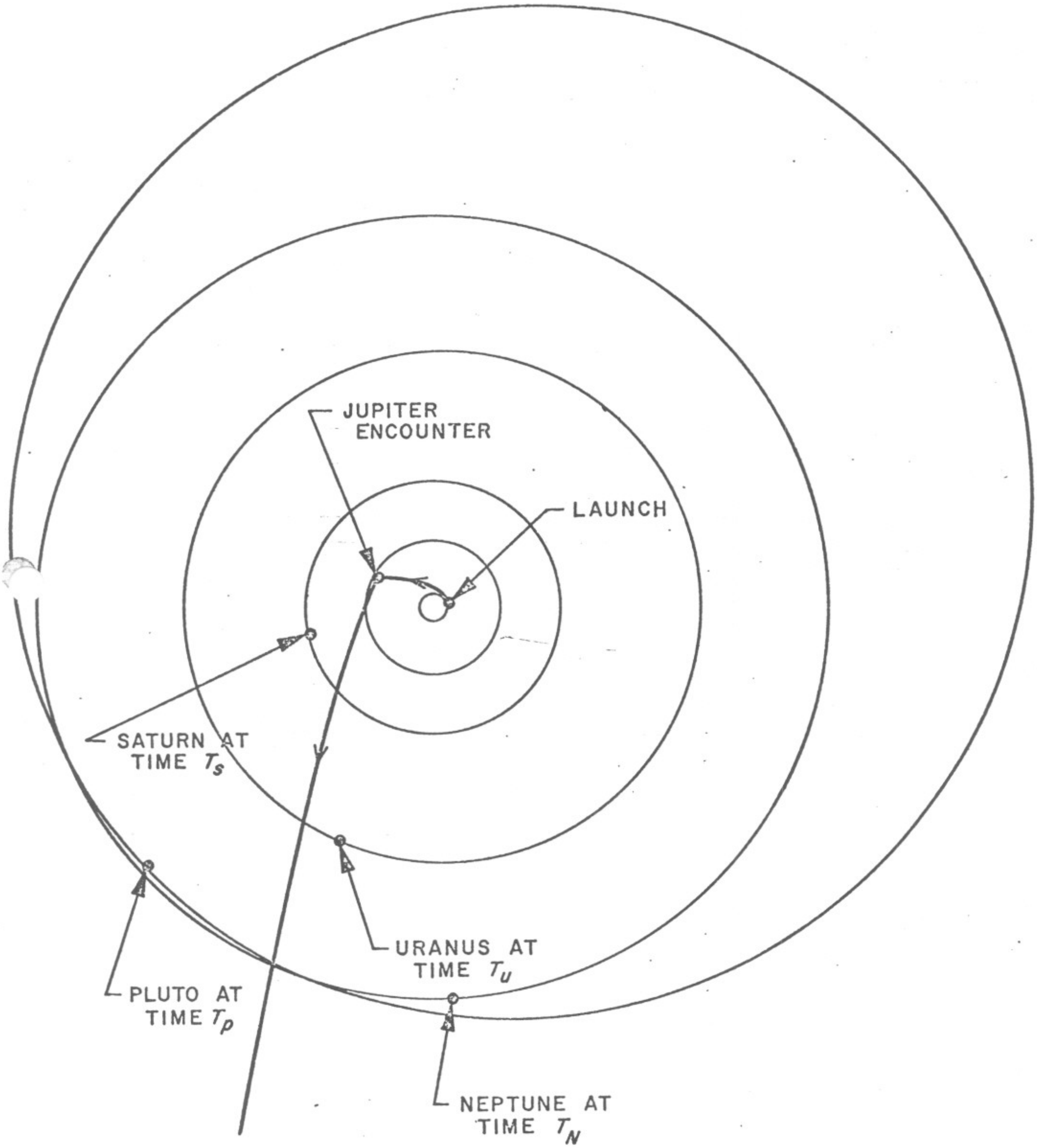

Figure 13. Trajectory profile for Earth-Jupiter-Deep Space 1978. The distance of closest approach is approximately 70,000 Km. The times T_s, T_u, T_n, and T_p represent the times when the probe passes closest to the orbits of the planets: Saturn, Uranus, Neptune and Pluto respectively

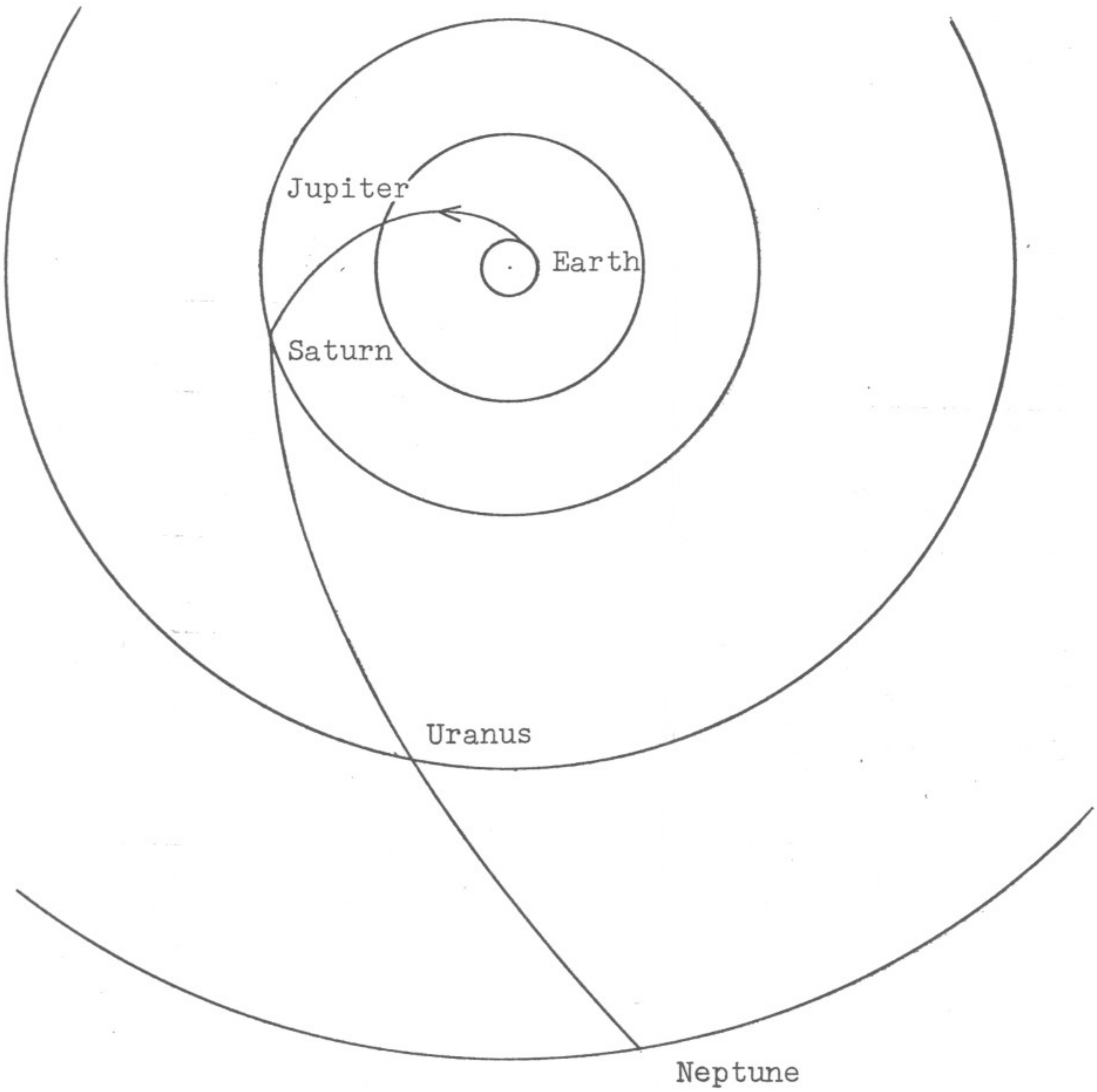

Figure 14. Trajectory profile for Earth-Jupiter-Saturn-Uranus-Neptune for t_o = September 2, 1977. Time to Neptune = 11.4 years.

figure suggests the possibility for obtaining a "grand tour" of all major planets (Jupiter, Saturn, Uranus, and Neptune) by one and the same space vehicle on a carefully calculated Gravity Thrust trajectory. These trajectories are indeed possible for all Earth-Jupiter minimum transfer opportunities during the period 1975 through 1980. An example is shown in figure 14. However, if this particular profile is not used before 1980, it may not present itself again for the next 175 years. Since Jupiter sweeps out 360° in its orbit about the Sun every 11.9 years, profiles such as Earth-Jupiter-Saturn, Earth-Jupiter-Uranus, Earth-Jupiter-Neptune, and Earth-Jupiter-Pluto all recur approximately every 11.9 years and last for two or three successive Earth-Jupiter launch opporunities. Various other profile combinations are also possible, though too numerous to mention in this work.

The environmental study of our solar system will not be complete until regions of space far from the ecliptic plane have been explored. Direct transfer trajectories into these regions will be the most difficult to achieve. For example, in order to obtain a 90° out-of-ecliptic orbit with semi-major axis 2.9 A.U. and eccentricity .74 an HEV of 48.5 km/sec would be required. Hence the initial mass required to send a 500 kgm instrument probe on this trajectory would be $500 \exp(48.5/4.2) = 5\times10^{7}$ kgm. A trajectory which could take a probe approximately the same distance out of

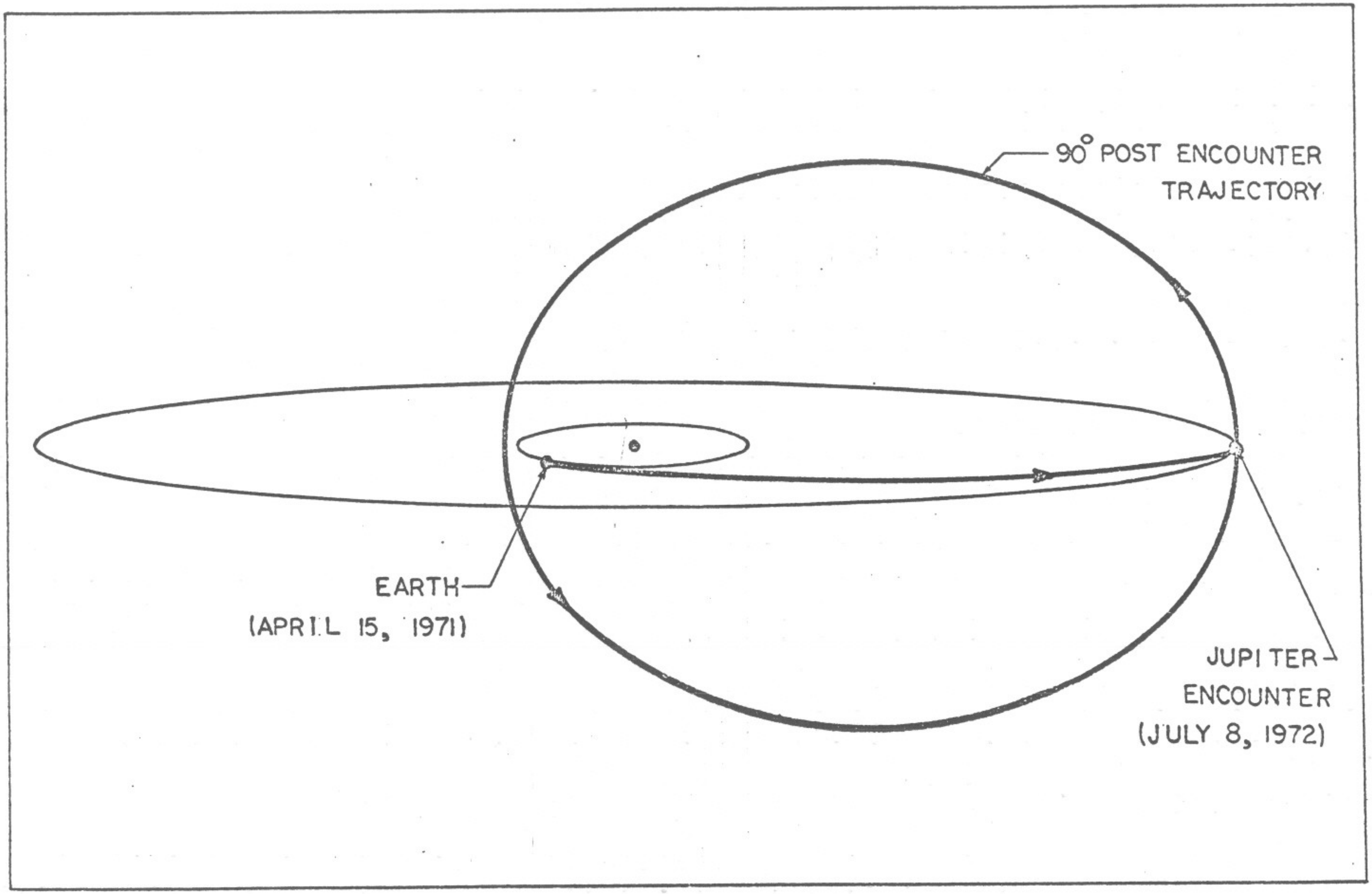

Figure 15. Trajectory profile for Earth-Jupiter-Out of Ecliptic 1971. The distance of closest approach is approximately 350,000 Km.

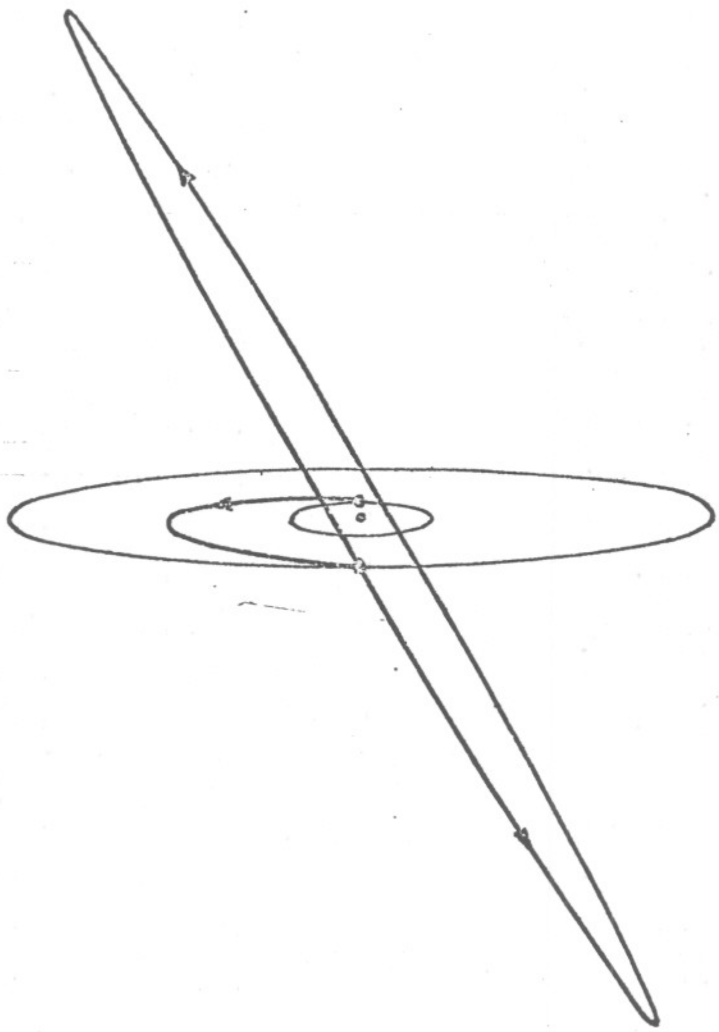

Figure 16. Trajectory profile for Earth-Jupiter-out of ecliptic. (This type is useful in attaining great distances from ecliptic plane.)

the ecliptic plane could be obtained by a relatively low energy Earth-Jupiter transfer and Gravity Thrust. The required HEV would only be about 11 km/sec. This would cut the required initial mass down to only 6,850 kgm (a reduction by a factor of 7.3×10^3). An example is described in figure 15. Regions further from the ecliptic plane can be obtained by increasing the energy of the post-encounter inclination a few degrees. An example is given in figure 16.

It is obvious from the above examples that the entire solar system could be economically explored via the Gravity Thrust which Jupiter could impart to a passing space vehicle. Therefore, since the minimum energy initial transfers to Venus and Mars require considerably less energy than minimum energy transfers to Jupiter, it is interesting to inquire into the possibility of reaching Jupiter via various Gravity Thrust profiles involving only Venus and Mars. Such possibilities do indeed exist. They are illustrated in figures 17 and 18. Various trajectory parameters corresponding to these examples are given in tables 1 and 2 respectively. In these tables the following notation is used:

ΔV = magnitude of velocity increase required for injection onto the initial transfer trajectory from a 200 km high circular Earth parking orbit (km/sec)

T = flight time between encounters (days)

θ = heliocentric transfer angle swept out about the Sun between encounters (degrees)

a = semi-major axis of interplanetary transfers between encounters

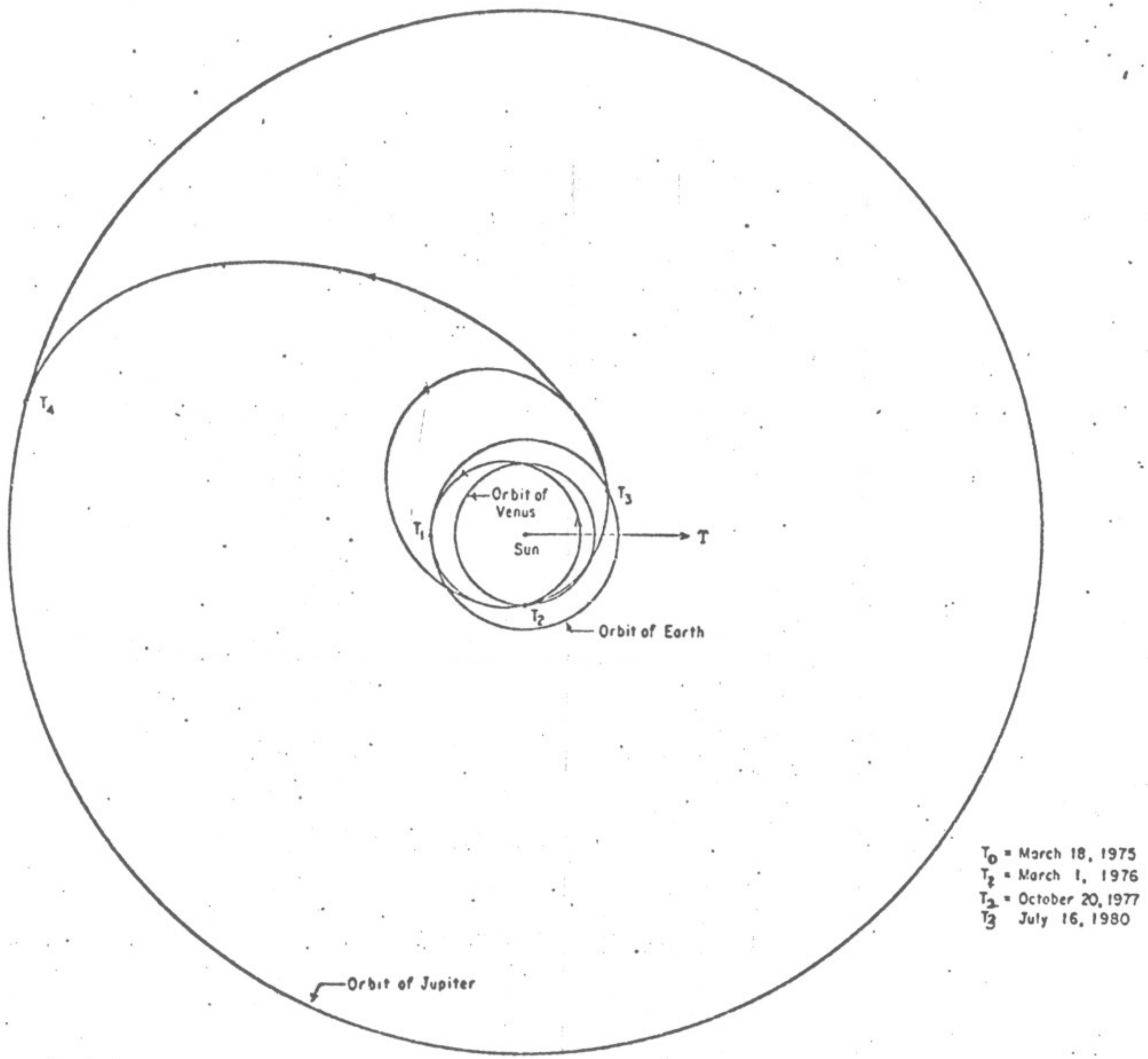

Figure 17. Earth - Venus - Earth - Jupiter

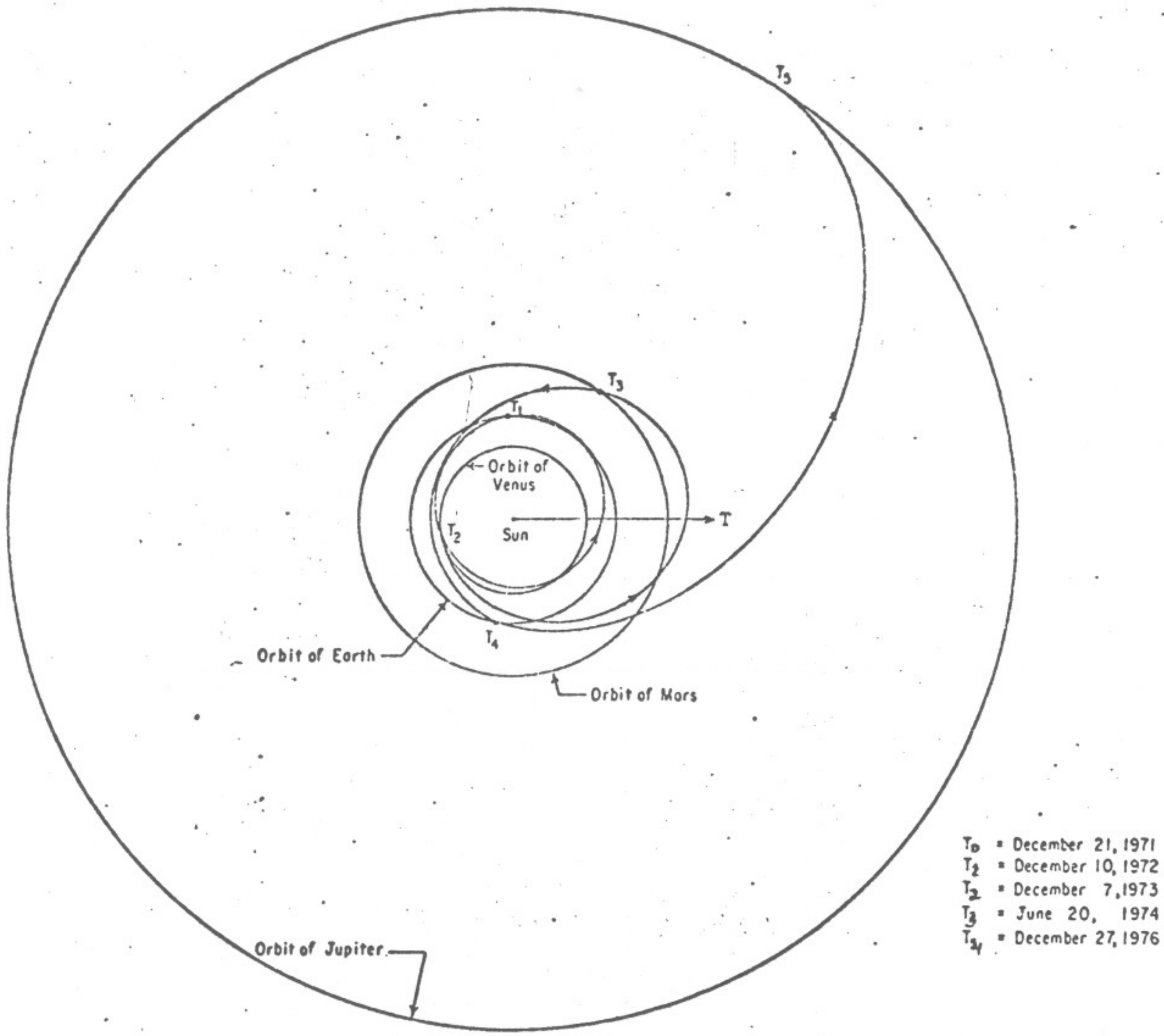

Figure 18. Earth - Venus - Mars - Earth - Jupiter

TABLE 1

ΔV= 4.06 km/sec

PLANET	DATE	HEV	T	Θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	3/18/75	4.14									
			349.6	457.1	0.795	0.252					
VENUS	3/1/76	9.06					669.0	1.55	43.3	5.8	6.69
			597.7	472.9	1.279	0.449					
EARTH	10/20/77	13.73					-421.8	1.54	30.4	6.9	7.19
			999.9	139.7	3.158	0.716					
JUPITER	7/16/80	5.73									

TABLE 2

ΔV= 4.10 km/sec

PLANET	DATE	HEV	T	Θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	12/21/71	4.25									
			354.0	457.8	0.818	0.214					
VENUS	12/10/72	6.96					264.8	1.96	61.7	8.0	7.14
			362.8	229.4	1.227	0.415					
MARS	12/ 7/73	11.11					356.6	1.17	9.7	4.4	1.88
			194.2	211.7	1.290	0.392					
EARTH	6/20/74	10.93					-218.4	1.96	41.2	8.7	7.68
			921.7	151.7	2.995	0.677					
JUPITER	12/27/76	5.98									

e = eccentricity of interplanetary transfers

DOCA = distance of closest approach to a planet's surface during an encounter (km)

TISI = time spent by the vehicle in a planet's activity region during an encounter (days)

DA = deflection angle of vehicle's velocity vector relative to the encountered planet

TDA = total deflection angle of vehicle's velocity vector relative to the Sun due to Gravity Thrust from an encountered planet

IMPV = magnitude of vehicle's velocity change due to Gravity Thrust

The required injection ΔV to obtain an HEV of 9.5 km/sec (which correspond to minimum energy Earth-Jupiter transfers) is about 6.6 km/sec. Hence, the profiles illustrated in figures 17 and 18 represent an initial mass reduction of approximately 85%. However, the flight time would be increased by about 2.8 years (about double the minimum energy Earth-Jupiter flight time). The negative distances of closest approach at Earth can be raised by either increasing the initial launch energy a small amount or by initiating a suitable velocity correction on the approach asymptote. Table 3 describes a single Gravity Thrust trajectory which will enable one space vehicle to pass by all the planets in the solar system except Mercury and Pluto. All the required distances of closest approach are positive. The trajectory

TABLE 3

ΔV = 4.87 Km/sec

PLANET	DATE	HEV	T	θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	12/31/71	6.25									
			335.3	433.0	0.826	0.258					
VENUS	11/30/72	8.68					2241.5	1.60	39.9	5.6	5.92
			376.7	247.3	1.230	0.435					
MARS	12/12/73	11.59					312.1	1.13	9.1	4.0	1.83
			190.5	209.9	1.312	0.406					
EARTH	6/20/74	11.31					92.9	1.89	37.9	8.2	7.35
			870.5	146.7	2.983	0.679					
JUPITER	11/7/76	6.02					284632.1	147.67	130.4	19.2	10.92
			1567.5	130.2	23.370	0.800					
SATURN	2/21/81	7.61					121396.3	150.96	130.0	43.9	11.91
			2154.0	77.6	8.157	2.169					
URANUS	1/15/87	11.15					186180.0	107.14	20.9	14.1	4.04
			1643.4	22.4	5.598	3.485					
NEPTUNE	7/16/91	12.79									

is in fact physically realizable, although the close encounter at Earth would require extremely accurate guidance. It should be noted that this truly "Grand Tour" requires an injection velocity of only 4.87 km/sec (generating a departing transfer trajectory with an HEV of 6.25 km/sec). The minimum energy direct transfer trajectory to Neptune requires an HEV of 11.62 km/sec and takes 31 years to complete. Of course the trajectory described in table 3 is rare, but the number of various profiles involving 7 planetary encounters is high.

4.3 Early Manned Interplanetary Exploration

Gravity Thrust trajectories of the form Earth-Venus-Earth could be utilized as the first manned interplanetary reconnaissance mission to another planet. The required HEV's for the initial Earth-Venus transfers vary from 3 to 3.8 km/sec, with total flight times of approximately one year. They are available every time the Earth-Venus launch window opens. The resulting distances of closest approach vary from 700 to 5000 km. Such missions would require the design and construction of special space vehicles suitable for extended manned flights of a year's duration. However, the required injection energies from an Earth parking orbit are so low, the required initial mass could be transported to the parking orbit for assembly by no more than two Saturn V type launch vehicles. Unfortunately, Gravity Thrust trajectories of the form Earth-Mars-Earth require total flight times of approximately 1000 days. However, this situation can be improved considerably by profiles of the form Earth-Venus-Mars-Earth. Although they are only attractive about 50% of the time the Earth-Venus window is open, they offer the opportunity for a reconnaissance of both Venus and Mars during the course of a single mission. The required HEV's of the initial transfer run from lows of 3.5 to highs of 4.2 km/sec. Total trip times run from 450 to 600 days. These profiles would be ideal for the first manned voyages

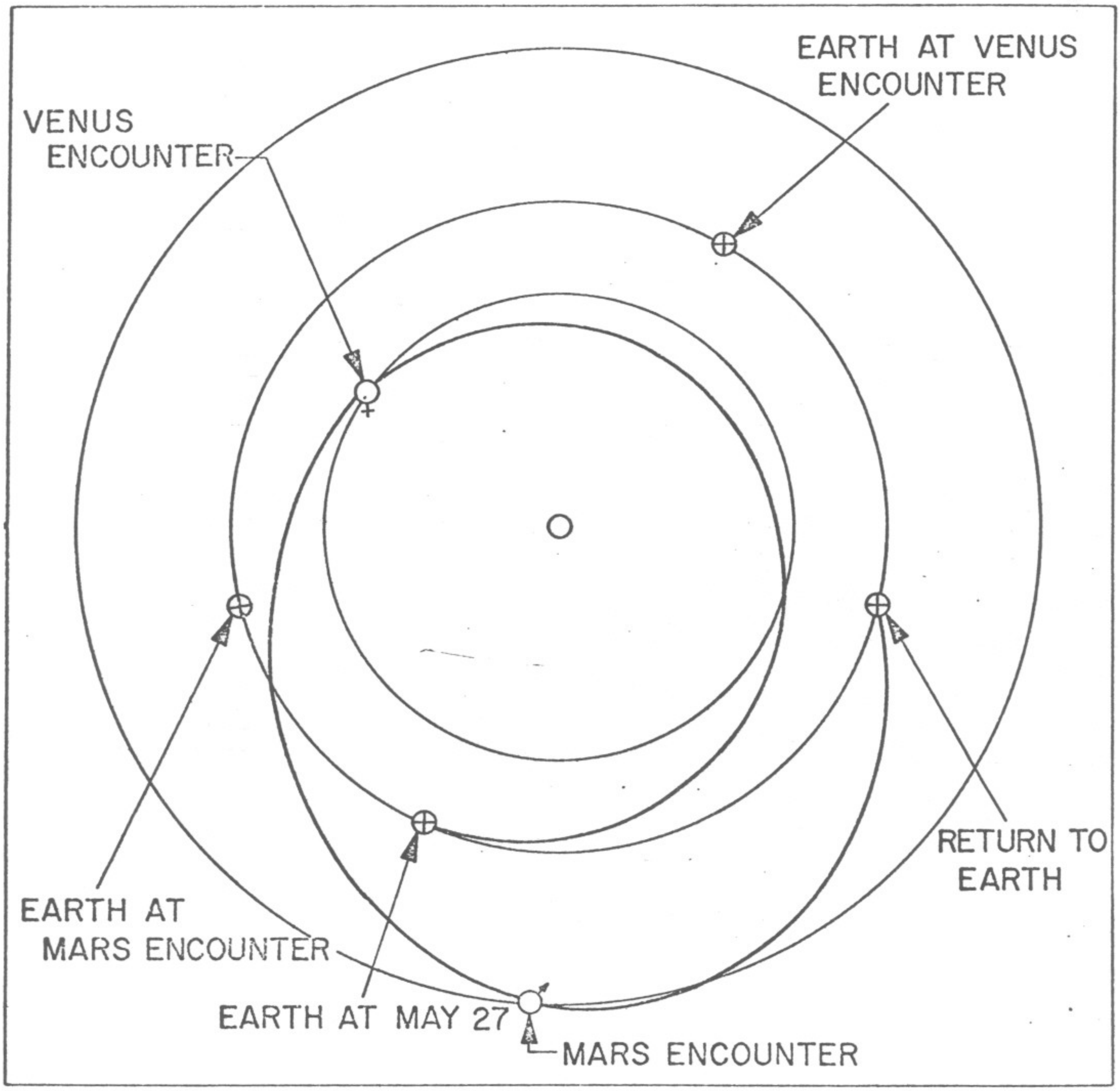

Figure 19. Trajectory profile for Earth-Venus-Mars-Earth 1972. The distances of closest approach to Venus and Mars are approximately 6,000 Km. and 1,000 Km. respectively. On the average each decade contains about 3 opportunities for these profiles, each lasting about one month in duration.

into interplanetary space. An example is given by figure 19. (It is interesting to note that the injection velocity corresponding to these profiles are about 4 km/sec. This is approximately the same ΔV Hohmann had to expend just in fighting the perturbations of Venus and Mars.)

Mars has traditionally been viewed as the prime target for the first manned landing expedition to another planet. Unfortunately, a minimum energy trajectory design for this mission would require a total trip time of about 1000 days. A shorter trip time using classical Earth-Mars, Mars-Earth transfers would require extremely high characteristic velocities. This dilemma is due to the fact that minimum energy Mars-Earth return transfers occur 40 to 60 days before the minimum energy Earth-Mars departing transfers. Hence, 500 to 600 days must be spent on Mars waiting for the minimum energy Mars-Earth transfer trajectories to return. Gravity Thrust trajectory design, however, can be judiciously, applied to significantly improve the picture. Instead of using the classical Earth-Mars departing transfer to travel to Mars, we can make use of the Earth-Venus-Mars profile. This will enable the space vehicle to reach Mars in time to return to Earth on the desired minimum energy Mars-Earth return trajectory, and there would still be time for an exploration period of 10 to 15 days on Mars. The actual mission profile could be based upon using two space vehicles, designated as vehicles A and B . Vehicle A is to be sent on a Gravity Thrust Earth-Venus-Mars-Earth trajectory and equipped with a small Apollo type Earth re-entry module. Vehicle B is to be sent on an Earth-Venus-Mars trajectory. In place of an Earth re-entry module as contained in A , this vehicle can be equipped with a small Mars excursion module similar to the lunar excursion module of project Apollo. However, unlike the lunar excursion module, this module is designed for nearly total aerodynamic breaking by the Martian atmosphere. This will enable its surface approach speed to be almost entirely eliminated by atmospheric drag rather than by the reverse thrust of an on-board rocket engine. Consequently, as B approaches Mars, the crew abandons their main transfer vehicle used for the interplanetary trip and makes the actual descent in the relatively small excursion module. The landing takes place about 10 days before A makes its closest approach, giving the crew of B about 8 or 9 days to explore the surface of Mars. Then, as A begins to make its closest approach, the small Mars excursion module is launched, leaving behind all weight not required to effect a successful rendezvous with A . The mission is completed in ship A . This mission profile is designed to take advantage of the fact that it requires much less energy to accelerate a relatively small Mars excursion module than it does to accelerate a large space vehicle. The basic tool, however, is Gravity Thrust. Theoretical studies (9) have shown that this mission can be accomplished by only two Saturn V type (non-nuclear) launch vehicles. Many other design profiles are possible utilizing Mars-Venus-Earth and Earth-Mars-Venus trajectories. The particular profile depends upon the particular launch dates considered. On the average, the use of Gravity Thrust has cut the total energy requirements for short term manned Mars exploration missions to about half their previous values (13).

4.4 Interplanetary Transportation Networks for Future Manned Space Travel

Assuming man finds the wisdom to control and direct the vast power technology provides, he will reach out into space beyond the moon and planets. This is as inevitable as was the landing on the moon. He will colonize the moon, and other planets and their moons along with asteroids. Before this can take place, a reasonable transportation system will have to be developed to enable man to make interplanetary voyages without having to destroy powerful and costly boosters in the process.

The "brute force" technique of designing a massive space ship which could transport great numbers of people from the surface of one planet to the surface of another would require so much energy that the concept borders on the realm of science fiction.

There are some obvious design techniques which could improve this situation. For example, the great mass of life support equipment required to keep the passengers comfortable during their long space voyage is superfluous during the relatively brief launching and landing phases. Much energy could be saved if all this equipment were left in orbit rather than carried with the passengers down to a planet's surface. Consequently the earliest space pioneers such as Goddard and Oberth suggested that the space liners should operate between planetary parking orbits instead of planetary surfaces. A subsystem of much smaller ferry vehicles would then be designed for each planet in the network to transport passengers between orbiting "space ports" in the parking orbits and various surface facilities. This would greatly reduce the propulsion requirements of the interplanetary space liners, and therefore make the concept at least feasible. However, since this design plan will require major propulsive maneuvers such as injections onto interplanetary trajectories and decelerations into parking orbits, the ship's propulsion system will still have to deal with the initial mass problem (which would be quite formidable since the ship's mass will be tremendous). Furthermore, all injections onto interplanetary transfer trajectories to a particular planet will have to be accomplished during a relatively short period when the required injection energies are at a minimum (unless, of course, one has engines of almost unlimited power, in which case our discussion becomes purely academic). The time intervals between these launch opportunities are equal to the launch planets' and arrival planets' synodic period. These time intervals can be quite long (780 days for transfers between Earth and Mars; 584 days for transfers between Earth and Venus). Hence the space ships in this network will have to spend long periods of time in various parking orbits, waiting for the desired launch opportunities -- more time, in fact, than would be required for many voyages.

I will now introduce (11) an interplanetary transportation network which will almost completely nullify the initial mass problem affecting the design of the main space ships and which will, at the same time, significantly improve the parking orbit waiting time situation. Before going into any details, I will first give a brief conceptual description of this network. The transition from the surface-to-surface concept to the parking orbit-to-parking orbit concept of network design was motivated by the fact that it is not necessary to bring the entire space ship down to a planet's surface for the passengers to reach it. The idea behind the network I shall now describe arises from the fact it is not even necessary to accelerate and decelerate the entire space ship in the process of transferring its passengers from an orbiting space port of one planet to an orbiting space port of another planet. If a second ferry vehicle subsystem can be designed for this purpose, the large space ships need not make any thrusting maneuver during a planetary encounter. A space ship can simply "coast" past a planet, using Gravity Thrust to direct it toward an encounter with another planet; the process can be repeated indefinitely. The ferry vehicles in the second subsystem (which I call "rendezvous modules") will be much smaller than the main space liners and will have a capacity of, perhaps, 50 passengers. Many of these modules can be used to transfer hundreds of passengers between various orbiting space ports and a single space liner during its encounter phase. The burden of accelerating this strictly passenger mass can thereby be distributed among the several rendezvous molecules. The entire network may involve several dozen massive space liners, all moving simultaneously on "never ending" journeys around the planets. This completes the basic conceptual description.

The actual construction of the interplanetary space liners will take place in vast orbiting "ship yards" equipped with all the facilities required to sustain hundreds of astronautical construction workers for weeks at a time. Since the ships will be designed for maximum passenger comfort, some form of artificial gravity will be desired. Hence the ship's overall shape can be spherical or toroidal (resembling a huge innertube). During its flight around the planets, a slow constant spin will then induce the desired artificial gravity environment. The ship's overall size will range from 100 to 1000 meters across. It will contain private living quarters, dining rooms, recreation areas, etc., -- in short, everything one might find on modern ocean linears, but scaled up so as to be adequate for journeys of one or two years. The ships will contain onboard repair facilities which will enable major repairs to be undertaken while they are far away from any planet. They will also be supplied with vast quantities of rocket fuel for the rendezvous modules. In short, these ships can be described as small, compact cities moving continuously from planet to planet.

When the construction of each ship is completed, preparations are made for its injection. All preparations prior to injection must be completed according to a strict time schedule to satisfy the demainding trajectory requirements. A space liner's interplanetary trajectory profile can be described by $P_0^j - P_1^j - P_2^j - \cdots$ where P_0^j = Earth and P_1^j = Venus or Mars. The symbol P_i^j refers to the i^{th} planet encountered by the j^{th} space liner in the network. The Gravity Thrust received from each successive planetary encounter P_i^j is the only thrust force used to propel the ship to the next planet P_{i+1}^j. Therefore, it is necessary to apply conventional rocket thrust (i.e., reaction thrust) only once, to inject the ship onto its initial transfer trajectory $P_0^j - P_1^j$. Thereafter the ship's guidance system takes over complete trajectory control. This initial transfer is chosen so that the required injection ΔV is an absolute minimum with respect to possible fixed trajectory constraints such as sufficiently great distances of closest approach at each successive planetary encounter. Preliminary calculations show that these required ΔV's will be approximately 3.7 km/sec or less. Hence it will be possible even for relatively weak second or third generation nuclear propulsion injection engines (with specific impulse in the neighborhood of 1,200 seconds) to launch relatively massive (first generation) space ships. Moreover, when the power of these injection engines is increased, the space liner's mass can be increased also, but by proportions hundreds of times greater than would be possible without Gravity Thrust. The injection will be extremely accurate, hence all subsequent trajectory corrections will be infinitesimal. The on-board rocket thrusts which will be required from time to time in order to maintain high trajectory accuracy can be assumed to be very small due to the high initial injection accuracy.

When the injection time approaches, passengers for the initial voyage board the ship with a full complement of supplies. Gigantic nuclear injection engines are brought into position and attached to the space ship. When the precisely calculated moment for injection occurs, the engines are started, sending the space liner on its perpetual journey around the planets. After the injection is accomplished, the engines are disconnected and sent back toward the Earth to rendezvous with another orbiting construction area for use in other injections.

As mentioned above, the complete network will consist of the main interplanetary space liners, a system of rendezvous modules operating between the space liners and various orbiting space ports, and a system of ferry vehicles operating between the space ports and surface facilities. It is natural to assume that the first planets to be colonized will be Mars and Venus. Thus if the first interplanetary transportation networks include only the Earth, Venus and Mars, the two secondary transportation subsystems can be designed to take maximum advantage of planetary atmospheres. The SCRAMJET (14) concept would be a most efficient design for the ferry vehicles providing transportation between various airports on the Earth's surface and various orbiting space ports. Furthermore, it does not seem unreasonable to assume that the SCRAMJET principle can be made to operate in other planetary atmospheres, such as those on Venus and Mars. Operation in the Martian atmosphere may, however, require difficult modifications, since that atmosphere is extremely tenuous.

The design of the rendezvous modules can also make use of planetary atmospheres. For example, after leaving a space liner with a load of passengers, a module could use atmospheric braking forces to provide almost all the energy required to slow it down before its rendezvous with an orbiting space port. This would greatly reduce the amount of fuel which would have to be supplied to the module by the space ship for this task. However, these modules will have to be equipped with the most powerful nuclear rocket engines in order to accelerate a load of passengers from an orbiting space port and rendezvous with a passing space liner. After completing the rendezvous, the modules can be placed on "automatic pilot" to keep them on an interplanetary trajectory very close to the space liner for use during future planetary encounters. Supplies for the main space liners can be taken aboard during each successive Earth encounter either by means of specially designed cargo modules or in large cargo compartments of the passenger rendezvous modules.

The most important aspects of the interplanetary transportation network described above are the trajectories used by the great space liners. The network will be only as good as the routes of the "interplanetary highways" it follows. Discovering the best highways will be no easy task. I will not enter into a technical discussion of multiple encounter, Gravity Thrust trajectories beyond what I have already presented in chapter 2. It is important, however, to point out the tremendous computational problems that must be dealt with when Gravity Thrust trajectories involving many successive planetary encounters are contemplated.

The initial conditions upon which a trajectory calculation is based are given by specifying the launch date t_o, the first planetary encounter date t_1, and the particular planets in the specified profile P_0-P_1-P_2- ··· -P_n, where n is any arbitrarily large integer. Successive encounters with the same planet (as for example - Mars-Venus-Venus-Earth-) are ruled out. Hence the total number of possible profiles involving n encounters where P_0 = Earth is 8^n. Since each space liner will be very expensive, and must remain in use for long periods of time, its trajectory must be determined for many years following its initial launch date t_o. The trajectory profile should be calculated as a unit, rather than in segments, because it may turn out that a given trajectory cannot extend beyond P_n without requiring unreasonably long flight times for the P_n-P_{n+1} leg. In other words, before each space ship can be launched its trajectory $P_0^j - P_1^j - \cdots - P_n^j$ must be calculated for relatively large n (depending upon the particular planets in the profile) with little option for subsequent change. For example, if the planets in the profile are restricted to the Earth, Venus and Mars, then n should be greater than 30 if the trajectory is to extend over a period of 30 years. Calculation of the network trajectories will be extremely difficult because the total number of encounters involved in any single profile trajectory may exceed 25 or 30, and the initial conditions do not uniquely determine the trajectory.

To illustrate the great computational problems one must face, let us suppose one wishes to analyze all possible profiles involving Earth, Venus and Mars with n encounters, given only P_0, t_o, P_1 and t_1. If all the heliocentric transfer angles θ, swept out about the sun between successive encounters are to be less than 540^o, each trajectory leg $P_i - P_{i+1}$ can have, in general, a maximum of 6 different intercept dates t_{i+1} at P_{i+1} which will have the same departing energy at P_i relative to P_i $(i \geq 2)$. Consequently, for each particular profile there may be a total of 6^{n-1} different trajectories satisfying the initial conditions. Thus the total number N(n) of different trajectories theoretically possible is

$$N(n) = 12^{n-1} .$$

For the case n = 30 this number becomes 9×10^{30}. Moreover, for a complete analysis corresponding to the given P_0-P_1 launch opportunity period a net (9) of trajectories with different t_0 and t_1 dates will have to be calculated. This net may involve 500 "points" (t_0,t_1) in the t_0,t_1 plane. This fact may raise the total number of possible trajectories to 4.5×10^{33}. Since several of these P_0-P_1 launch opportunities may have to be investigated, one is left with a number of different possible trajectories enormous beyond comprehension. The cost of computing all these trajectories would be greater than the total construction costs of the transportation network itself!

Although a "brute force" numerical attack on this problem is impossible, special computing techniques will automatically reject unfavorable trajectories before they are actually calculated. During the early stages of computation, these special techniques monitor a few critical factors upon which the characteristics of significant portions of possible trajectories depend. With the aid of these techniques, only an infinitesimal fraction of the total number of possible trajectories need be seriously investigated. This drastically reduced range of possibilities still presents a formidable computational task, however. In the above case where $n = 30$ the problem must be processed on a computer at least as powerful as the CDC 6600.

The trajectories shown in Tables 4 through 8 were calculated at the University of California Computer Facility at Berkeley on a CDC 6400 (15). These trajectories are intended to show that the main space liners in the interplanetary transportation network described in this work can in fact be propelled around the Solar System solely by Gravity Thrust. Of course, since these examples are not the results of any large scale effort to compute a suitable trajectory network, they should be viewed as poor choices for such a network.

Table 4 is an example of a trajectory designed to fly a space liner back and forth continuously between Earth and Venus. The average flight time between encounters is 333 days, or 251 days less than the synodic period of these planets. Other trajectories of this type also have this property. Therefore, a combination of several such trajectories will significantly reduce the waiting period between flight opportunities, which is often very long in parking-orbit-to-parking-orbit network designs. Notice that since the required ΔV is extremely small an uprated Saturn V launch vehicle will be able to inject a 62,000 kgm vehicle onto this trajectory. It should be easy to construct a modest 5-man "first generation" interplanetary space ship of approximately this payload mass, since no major propulsive maneuvers will ever be required. The first few encounters can be used for manned Venus fly-by missions. Later on, when several ships of this type are sent on early network trajectories, they could play an important role in the design of early manned planetary landing expeditions.

Table 5 is an example of a profile which will take a space ship around three planets -- Earth, Venus and Mars. In this case the average flight time between encounters is 374 days. With the aid of a neclear rocket injection engine (Isp = 850 seconds) capable of injecting a 586,000 kgm payload from an initial orbital mass of 900,000 kgm, a half-million kgm space vehicle can be injected onto the first leg of this network trajectory. If an "Orion" nuclear pulse injection engine (Isp = 2500 seconds) is used, the payload can be increased to 775,000 kgm. With this mass available, the vehicle can begin to take the shape of a true space liner. A trajectory profile involving 16 planetary encounters is shown in Table 6. The first four legs of this trajectory are almost identical to the trajectory in Table 4. Notice that the flight time for the eleventh leg is 803 days. In a complete network employing several dozen space liners, there will probably be one liner on a trajectory passing Venus within a few weeks of September 27, 1984, and encountering Mars with a transfer time of only 250 to 300 days. Passengers wishing to go to Mars from Venus will take this ship rather than the ship on the Table 6 trajectory. An Example of a possible trajectory of such an alternative space liner is shown in Table 7. The last example, illustrated in Table 8, is intended to show that a single interplanetary transportation network can connect the inner planets with the outer planets -- it is not necessary to construct two fundamentally different networks to serve the two planetary groups. The individual space liners will be able to operate among both the inner and the outer planets. However, encounters involving Saturn, Uranus, Neptune or Pluto will generally require very long flight times (i.e., more than 5 years between encounters).

TABLE 4

ΔV = 3.67 km/sec

PLANET	DATE	HEV	T	Θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	11/1/73	3.18									
			154.0	198.4	0.855	0.160					
VENUS	4/4/74	5.16					18844.4	2.63	33.4	3.9	3.39
			299.4	251.0	1.016	0.288					
EARTH	1/29/75	9.03					19637.1	2.29	18.2	2.5	2.86
			342.5	417.4	0.868	0.312					
VENUS	1/6/76	10.54					8855.0	1.33	18.8	2.7	3.45
			392.3	307.1	1.117	0.410					
EARTH	2/1/77	13.01					18270.0	1.61	10.0	3.3	2.27
			551.7	485.4	1.194	0.424					
VENUS	8/7/78	9.35					9095.9	1.50	22.6	3.4	3.67
			372.5	423.5	0.916	0.308					
EARTH	8/15/79	8.85					18685.0	2.40	19.4	5.7	2.99
			250.8	215.3	0.968	0.277					
VENUS	4/21/80	6.87					60121.7	2.01	10.8	1.6	1.29
			467.4	492.0	1.026	0.316					
EARTH	8/2/81	9.95					13491.6	2.15	19.4	4.6	3.35
			257.6	308.5	0.881	0.361					
VENUS	4/16/82	12.18					59971.4	1.16	3.7	0.6	0.78
			223.1	164.9	0.930	0.376					
EARTH	11/25/82	10.96					57888.0	1.91	5.6	2.1	1.08
			359.3	407.2	0.917	0.377					
VENUS	11/20/83	12.27					6501.6	1.14	16.8	3.11	3.59
			569.6	510.3	1.210	0.476					
EARTH	6/11/85	14.43					17770.5	1.49	8.4	2.5	2.12
			307.1	173.9	1.065	0.476					
VENUS	4/14/86	14.97					4305.3	.94	14.0	2.4	3.66
			117.2	242.4	0.816	0.418					
EARTH	8/10/86	10.71									

TABLE 5

$\Delta V = 3.65$ Km/sec

PLANET	DATE	HEV	T	θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	5/14/75	3.09									
			162.0	191.5	0.869	0.170					
VENUS	10/23/75	5.16					16814.3	2.62	40.7	4.7	3.59
			276.5	238.7	0.994	0.277					
EARTH	7/26/76	7.99					32634.8	2.67	15.8	2.6	2.20
			362.1	424.7	0.892	0.302					
VENUS	7/23/77	9.39					1404.8	1.49	38.4	6.2	6.18
			495.1	261.6	1.383	0.493					
MARS	11/30/78	10.63					417.0	1.20	10.4	4.3	1.92
			606.9	396.9	1.314	0.500					
EARTH	7/29/80	14.82					8225.7	1.46	12.7	5.2	3.28
			535.5	525.4	1.144	0.473					
VENUS	1/16/82	15.11					2535.6	0.93	16.3	5.8	4.27
			400.7	399.5	1.006	0.452					
EARTH	2/20/83	13.74					4519.8	1.53	18.7	6.3	4.46
			441.9	230.4	1.260	0.466					
VENUS	5/7/84	10.81					247412.8	1.20	1.2	0.3	0.23
			470.8	306.5	1.249	0.463					
EARTH	8/21/85	13.66					7484.9	1.57	15.4	5.3	3.65
			318.1	316.6	1.047	0.456					
MARS	7/5/86	7.91					558.6	1.58	17.0	5.1	2.33
			109.6	96.3	1.125	0.424					
VENUS	10/23/86	10.85					10077.3	1.30	16.8	3.9	3.16
			301.2	305.4	0.964	0.358					
EARTH	8/20/87	10.34									

TABLE 6

ΔV= 3.67 km/sec

PLANET	DATE	HEV	T	Θ	a	e	DOCA	TISI	DA	TDA	IMPY
EARTH	11/1/73	3.18									
			154.0	198.4	0.855	0.160					
VENUS	4/4/74	5.16					18844.4	2.63	38.4	3.9	3.39
			299.4	251.0	1.016	0.288					
EARTH	1/29/75	9.03					19637.1	2.29	18.2	2.5	2.86
			342.5	417.4	0.868	0.312					
VENUS	1/6/76	10.54					8855.0	1.33	18.8	2.7	3.45
			392.3	307.0	1.117	0.410					
EARTH	2/1/77	13.01					75791.8	1.61	3.2	0.5	0.73
			510.0	418.7	1.180	0.418					
VENUS	6/26/78	9.29					5379.4	1.50	28.6	5.8	4.59
			443.2	518.0	0.957	0.289					
EARTH	9/13/79	9.80					3286.7	2.16	34.9	8.0	5.89
			633.8	422.1	1.305	0.354					
MARS	6/7/81	6.34					2605.7	2.03	17.4	2.3	1.91
			329.2	232.4	1.177	0.391					
VENUS	5/3/82	7.16					900.2	1.94	56.8	9.5	6.80
			537.5	464.4	1.183	0.391					
EARTH	10/22/83	12.14					38565.3	1.74	6.5	2.0	1.38
			340.9	223.4	1.105	0.383					
VENUS	9/27/84	9.33					5196.2	1.51	28.8	5.2	4.63
			802.8	490.6	1.536	0.534					
MARS	12/9/86	12.69					1936.1	0.98	5.4	1.5	1.21
			458.3	148.7	1.412	0.535					
EARTH	3/11/88	16.37					14789.9	1.29	7.5	2.3	2.15
			510.4	399.1	1.200	0.508					
VENUS	8/4/89	15.38					122.0	0.92	20.8	3.8	5.56
			216.6	317.4	0.789	0.412					
EARTH	3/8/90	10.79					1179.2	1.94	36.3	2.6	6.73
			197.7	220.3	1.197	0.357					
MARS	9/22/90	6.76					2891.6	1.84	14.9	2.9	1.75
			302.8	250.8	1.109	0.391					
VENUS	7/22/91	9.35									

TABLE 7

$\Delta V = 3.67$ km/sec

PLANET	DATE	HEV	T	Θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	11/1/73	3.18									
			154.0	193.4	0.855	0.160					
VENUS	4/4/74	5.16					18844.4	2.63	38.4	3.9	3.39
			299.4	251.0	1.016	0.288					
EARTH	1/29/75	9.03					19637.1	2.29	18.2	2.5	2.86
			342.5	417.4	0.868	0.312					
VENUS	1/6/76	10.54					8855.0	1.33	18.8	2.7	3.45
			392.3	307.0	1.117	0.410					
EARTH	2/1/77	13.01					75791.8	1.61	3.2	0.5	0.73
			510.0	418.7	1.180	0.418					
VENUS	6/26/78	9.29					5379.4	1.50	28.6	5.8	4.59
			443.2	518.0	0.957	0.289					
EARTH	9/13/79	9.80					3286.7	2.16	34.9	8.0	5.89
			633.8	422.1	1.306	0.354					
MARS	6/7/81	6.34					2334.6	2.03	18.1	2.5	1.99
			307.8	198.4	1.173	0.387					
VENUS	4/11/82	7.53					57665.0	1.85	9.5	1.7	1.24
			531.1	470.9	1.165	0.387					
EARTH	9/24/83	12.06					37365.9	1.76	6.8	2.5	1.42
			396.0	293.9	1.139	0.382					
VENUS	10/24/84	8.11					7770.7	1.73	30.4	5.9	4.26
			275.0	184.6	1.197	0.394					
MARS	7/26/85	6.25					5542.4	2.28	12.6	4.2	1.37
			617.6	435.4	1.205	0.394					
EARTH	4/5/87	12.17					18243.7	1.74	11.3	2.6	2.40
			425.6	412.6	1.035	0.382					
VENUS	6/4/88	11.07					11957.4	1.28	14.7	3.0	2.83
			371.1	250.2	1.255	0.467					
MARS	6/10/89	8.71					3800.5	1.66	8.4	2.0	1.27
			672.0	424.9	1.334	0.428					
EARTH	4/13/91	12.55					53391.0	1.69	4.7	0.9	1.02
			551.9	440.1	1.228	0.409					
VENUS	10/16/92										

TABLE 8

ΔV= 4.92 km/sec

PLANET	DATE	HEV	T	Θ	a	e	DOCA	TISI	DA	TDA	IMPV
EARTH	12/30/71	6.34									
			335.0	432.2	0.826	0.260					
VENUS	11/29/72	8.74					2318.2	1.59	39.2	5.6	5.87
			376.2	247.9	1.229	0.435					
MARS	12/10/73	11.60					262.0	1.13	9.2	3.9	1.85
			189.9	208.9	1.315	0.405					
EARTH	6/18/74	11.22					95.5	1.91	38.3	8.2	7.37
			840.0	145.6	2.994	0.679					
JUPITER	10/5/76	6.06					236356.6	146.50	33.3	23.3	11.13
			1590.1	132.8	22.034	0.789					
SATURN	2/11/81	7.54					762688.5	153.23	53.2	12.6	6.75
			3609.8	273.9	5.72	0.901					
EARTH	12/31/90	25.80					8526.2	0.81	4.4	2.0	1.99
			624.2	75.8	3.29	0.843					
JUPITER	9/15/92	8.73					969653.9	121.14	75.5	88.1	10.75
			631.7	82.9	3.36	0.819					
EARTH	6/9/94	23.29									

Appendix

The Calculation of Planetary Position and Velocity Vectors

The process of computing Gravity Thrust space trajectories will involve the computation of many different planetary position and velocity vectors spanning large time intervals (e.g. over 30 years). The standard techniques usually involve various numerical interpolation and numerical differentiation procedures. These methods use blocks of tabulated position vectors covering a relatively small time interval centered around the specific time at which the vectors are to be computed. The techniques are extremely accurate. However, if one desires the capability of computing these vectors for all the planets for any arbitrary time within a long time interval, the required computer storage needed for the tables and interpolation and numerical differentiation programs will be large. If this program, along with the required Tables, are placed in the computer's high speed memory unit, little room (if any) will be left for the main program. If the main program is large, the subprogram which computes these planetary position and velocity vectors will have to be stored outside of the computer's central processing unit (CPU). However, each time these vectors are needed in the main program, the computer will have to reach outside its CPU. This constant process of going in and out of a computer's high speed memory unit can slow down the numerical computations by a factor of 10,000. To alleviate this problem I shall present a relatively simple method for computing planetary position and velocity vectors over extended time intervals which requires relatively little total storage area. By comparing many numerical results with those obtained by standard methods, it was observed that very little accuracy was lost.

Let t be any time within a long time interval $[T_1^*, T_2^*]$ (where $T_2^* - T_1^* \approx 30$ years). Let $\vec{R}_p(t)$ and $\vec{V}_p(t)$ correspond to a planet's position and velocity vectors at t respectively which are to be calculated. Suppose $\vec{R}_p(T_1)$ and $\vec{R}_p(T_2)$ are two tabulated position vectors of the planet such that $T_1 \leqq t \leqq T_2$. The time interval, $T_2 - T_1$, is chosen so that $\measuredangle\, \vec{R}_p(T_1), \vec{R}_p(T_2) < 180^\circ$. These time intervals corresponding to the various planets are given in the following Table:

Planet	T_2-T_1 (days)	Number of Entries
Mercury	20	550
Venus	100	110
Earth	150	75
Mars	300	40
Jupiter	500	22
Saturn	500	22
Uranus	500	22
Neptune	500	22
Pluto	500	22

The column labeled "number of entries" refers to the total number of tabulated position vectors used to span a 30 year time period. Hence, a total of only 885 positions vectors are required for the entire table, which covers a period of 30 years for all nine planets.

The calculation of $\vec{R}_p(t)$ and $\vec{V}_p(t)$ begins by computing the planet's osculating orbital vectors $\vec{e}$ and $\vec{h}$ from $\vec{R}_p(T_1)$ and $\vec{R}_p(T_2)$. This is easily carried out by the method described in section 1.4. For each planet the corresponding Lambert function is given by (1.2.2). The solution of the osculating semi-major axis a is almost immediately obtained, since semi-major axes of all planets are already approximately known. Let $\vec{D} = \vec{R}_p(T_2) - \vec{R}_p(T_1)$. Then, for some scalar factor f, $\hat{R}_p(t)$ can be expressed as

$$\hat{R}_p(t) = \frac{\vec{R}_p(T_1) + f\,\vec{D}}{|\vec{R}_p(T_1) + f\,\vec{D}|} \quad . \tag{A-1}$$

Clearly, $0 \leqq f \leqq 1$. We define d by

$$f = \frac{d}{D} .$$

Then, by referring to figure 20, we find

$$\frac{d}{\sin\delta} = \frac{R_p(T_1)}{\sin(\beta + \eta)}$$

and

$$\frac{D - d}{\sin\eta} = \frac{R_p(T_2)}{\sin(\beta + \eta)} .$$

Consequently, it follows that

$$f = \left[1 + \frac{R_p(T_2)\,\sin(\sigma - \delta)}{R_p(T_1)\sin\delta} \right]^{-1}, \tag{A-2}$$

where $\sigma = \delta + \eta$. This angle can be easily calculated by

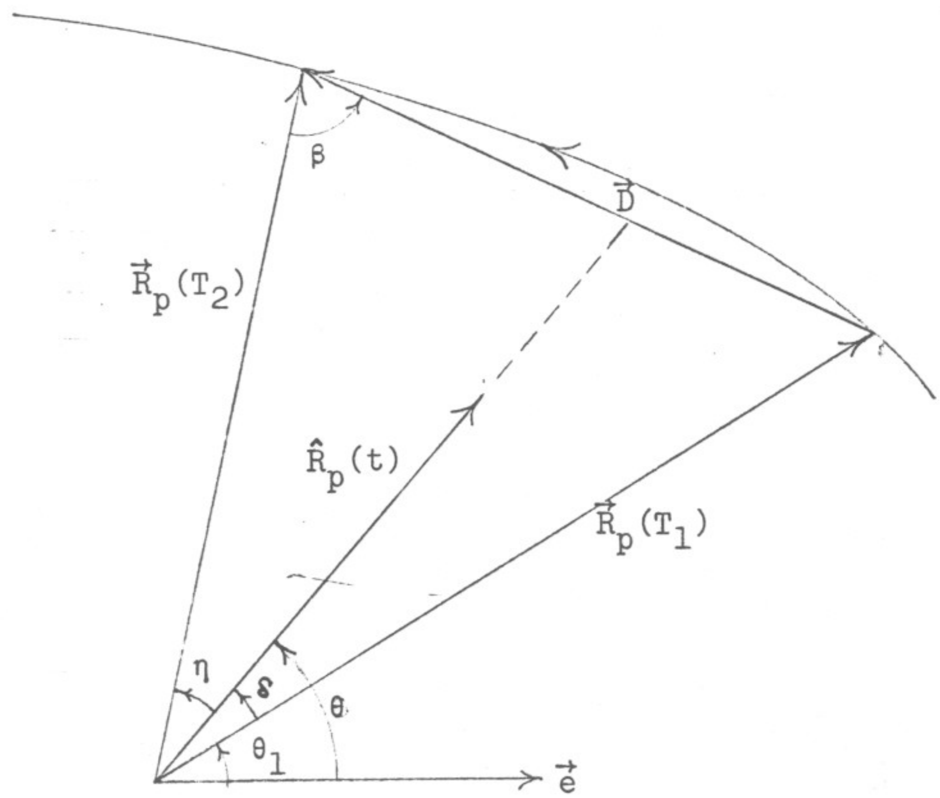

Figure 20. Geometry for calculating a planet's position vector

$$\sigma = \cos^{-1}\left[\frac{\vec{R}_p(T_1)\cdot\vec{R}_p(T_2)}{R_p(T_1)\ R_p(T_2)}\right] .$$

Now, the angle δ is dependent upon some scalar function $F(t)$. Since $T_2 - T_1$ is relatively small compared to the corresponding planet's orbital period, $F(t)$ can be expressed as a Taylor series about T_1 :

$$\delta = F(T_1) + \left(\frac{dF}{dt}\right)_{T_1}(t-T_1) + \frac{1}{2}\left(\frac{d^2F}{dt^2}\right)_{T_1}(t-T_1)^2 + \cdots . \qquad \text{(A-3)}$$

Since $\delta = 0$ when $t = T_1$, it follows that $F(T_1) = 0$. Let $\theta = \measuredangle\, \vec{e}, \vec{R}_p(t)$ and $\theta_1 = \measuredangle\, \vec{e}, \vec{R}_p(T_1)$ so that $360^o > \theta - \theta_1 \geqq 0$. Let t_o correspond to the planet's time of perihelion passage, where $T_p > t > t_o > 0$ (T_p is the planet's period). Hence,

$$\delta = F(t) = \theta - \theta_1 .$$

Now, from Kepler's second law of planetary motion, we may write

$$t-t_o = \frac{1}{\ell h}\int_0^\theta R^2 d\theta .$$

Therefore,

$$\left(\frac{dF}{dt}\right)_{T_1} = \frac{h}{\ell}(1 + e\cos\theta_1)^2$$

$$\left(\frac{d^2F}{dt^2}\right)_{T_1} = -\ 2\left(\frac{h}{\ell}\right)^2 e(1 + e\cos\theta_1)^3 \sin\theta_1$$

and

$$\left(\frac{d^3F}{dt^3}\right)_{T_1} = 2\left(\frac{h}{\ell}\right)^3 e(1 + e\cos\theta_1)^4[3\ e\sin^2\theta_1 - (1 + e\cos\theta_1)\cos\theta_1] .$$

For all planets except Mercury, the Taylor series converges very rapidly, so that all terms except the first three can be neglected. In the case of Mercury, the fourth term does contribute a small but sometimes nonnegligible amount to the series. The maximum contribution of this term has been found to be about 0.2 degrees (i.e., when $T_2 - T_1 \approx 20$ days). Thus, we shall include the fourth term in the Taylor expansion of δ. This term is

$$\left(\frac{d^4F}{dt^4}\right)_{T_1} = 2e\left(\frac{h}{\ell}\right)^4 \sin\theta_1(1 + e\cos\theta_1)^5[24e^2\cos^2\theta_1 + 13e\cos\theta_1 - 12e^2 + 1] .$$

The values of $\cos\theta_1$ and $\sin\theta_1$ can be computed from

$$\cos\theta_1 = \frac{\vec{R}_p(T_1)\cdot\vec{e}}{R_p(T_1)e}$$

and

$$\sin\theta_1 = S(1-\cos^2\theta_1)^{1/2} ,$$

where $S = \pm 1$ and computed from

$$S = \frac{(\vec{e}\times\vec{R}_p(T_1))\cdot\vec{h}}{|(\vec{e}\times\vec{R}_p(T_1))\cdot\vec{h}|} .$$

Consequently, after calculating each term, δ can be obtained by (A-3). Upon substituting this value into equation (A-2), f can be calculated. The unit vector $\hat{R}_p(t)$ can then be calculated by (A-1). The required position vector $\vec{R}_p(t)$ is obtained by

$$\vec{R}_p(t) = \left[\frac{\ell}{1+\vec{e}\cdot\hat{R}_p(t)}\right]\hat{R}_p(t) .$$

The corresponding velocity vector can now be immediately calculated by

$$\vec{V}_p(t) = \vec{h} \times [\vec{e} + \hat{R}_p(t)] .$$

The total computer storage space required for this program and the shortened planetary ephemeris is sufficiently small so that it can be incorporated into the main Gravity Thrust trajectory program (corresponding to section 2.4) and still fit into a 32K computer. This will allow our general Gravity Thrust trajectories to be computed as an entire trajectory without ever having to reach outside the computer's CPU to pull in any necessary planetary data.

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